Sketching Polynomial Graphs

1. Sketch a graph of the polynomial function \( f \) if
   \[
   f \text{ is increasing when } x < -1 \text{ and } 0 < x < 1,
   \]
   \[
   f \text{ is decreasing when } -1 < x < 0 \text{ and } x > 1,
   \]
   and \( f(x) < 0 \) for all real numbers.
   
   Describe the degree and leading coefficient of the function \( f \).

2. \( f \) is increasing on the interval \((-2, 3)\); \( f \) is decreasing on the intervals \((-\infty, -2)\) and \((3, \infty)\).

3. \( f(x) > 0 \) on the intervals \((-\infty, -4)\) and \((1, 5)\);
   \( f(x) < 0 \) on the intervals \((-4, 1)\) and \((5, \infty)\).

4. The number of students \( S \) (in thousands) who graduate in four years from a university can be modeled by the function \( S(t) = -\frac{1}{4}t^3 + t^2 + 23 \), where \( t \) is the number of years since 2010.
   
   a. Use a graphing calculator to graph the function for the interval \( 0 \leq t \leq 5 \). Describe the behavior of the graph on this interval.

   b. What is the average rate of change in the number of four-year graduates from 2010 to 2015?

   c. Do you think this model can be used for years before 2010 or after 2015? Explain your reasoning.