RATEY – A Graphing Organizer

RATEY is an organizing tool for graphing rational functions. To use this tool effectively, the numerator and the denominator should be factored completely over the real numbers before beginning the graph. RATEY is an acronym for the following:

**R** is for **Roots (zeros) and Removable discontinuities.**

Factor the numerator and denominator. Identify common factors and label them as removable discontinuities (holes in the graph). Simplify the expression, set the numerator equal to 0, and solve.

Graph these point(s) on the $x$-axis.

**A** is for **Asymptotes.** (Vertical only)

Using the simplified expression, set the denominator equal to 0 and solve.

Sketch the vertical asymptotes as dotted lines.

**T** is for **Two and means Two things.**

Do any of the factors of $R$ and $A$ have an even exponent (a multiple of 2) such as $(x-1)^2$ or $(x-1)^4$?

In the numerator, this means “T”angency at the root.

In the denominator, this means “T”ogetherness about the asymptote.

**E** is for **End Behavior.**

Determine the limit of the function that models the end behavior as $x \rightarrow \pm \infty$.

Compare the degree of the numerator and the denominator.

- If the numerator and denominator have the same degree, divide the coefficient of the largest degree term in the numerator by the coefficient of the largest degree term in the denominator to determine the horizontal asymptote.
- If the degree of the denominator is larger than the degree of the numerator, the end behavior asymptote is $y = 0$.
- If the degree of the numerator is larger than the degree of the denominator, perform long division to determine the oblique asymptote.

Sketch the asymptotes for the end behavior as dotted lines.

**Y** is for **Y-Intercept.**

Let $x$ equal zero and solve for $y$.

Graph this point on the $y$-axis.
For questions 1-8, identify the RATEY characteristics and sketch the graph:

1. \( f(x) = \frac{x - 4}{x + 2} \)

2. \( f(x) = \frac{(x + 1)(x - 3)}{x + 2} \)

3. \( f(x) = \frac{x^2 + x - 6}{(x + 1)^2} \)
4. \[f(x) = \frac{2x(x-1)^2}{(x+1)^3}\]

5. \[f(x) = \frac{-3(x-1)^2}{(x+5)^2}\]

6. \[f(x) = \frac{9-x}{x^2-4}\]
7. \[ f(x) = \frac{(x^2 + 4x + 8)(x - 4)}{4(x^2 - 4x)} \]

8. \[ f(x) = \frac{2x^2 - 3x - 9}{x} \]

9. Describe the similarities and differences between the graphs of \( f(x) = \frac{6x - 6}{2x - 3} \) and \( g(x) = \frac{2x - 3}{6x - 6} \).
10. Write a possible equation for each of the functions in these graphs. Compare a calculator’s graph of your equation to the given graph.

a. 

b. 

c. 

\[
\begin{align*}
&f(x) = 1 - x - \frac{1}{x} \\
&f(x) = -\frac{1}{x} - 1
\end{align*}
\]
11. Given \( f(x) = \frac{ax + b}{b - ax} \):

a. What is the root of \( f(x) \)?

b. What is the equation of the function’s vertical asymptote?

c. What is the end behavior of the function?

d. What is its \( y \)-intercept?

e. If \( a > b > 0 \), sketch \( f(x) \).

f. On what intervals does \( f(x) \) appear to be increasing?

g. On what intervals does \( f(x) \) appear to be concave up?