Exercises 2.7 Logarithms and Exponentials

1. Evaluate
   (a) \( \log_{10} 1000 \)
   (b) \( \log_4 1 \)
   (c) \( \log_3 27 \)
   (d) \( \log_2 \frac{1}{4} \)
   (e) \( \log_a a^x \)

2. Solve for \( x \)
   (a) \( \log_4 x = 2 \)
   (b) \( \log_{\frac{1}{8}} x = 4 \)
   (c) \( \log_{10} (2x + 1) = 2 \)
   (d) \( \log_2 64 = x \)
   (e) \( \log_b 81 = 4 \)

3. (a) Use log laws to solve \( \log_3 x = \log_3 7 + \log_3 3 \).
   (b) Without tables, simplify \( 2 \log_{10} 5 + \log_{10} 8 - \log_{10} 2 \).
   (c) If \( \log_{10} 8 = x \) and \( \log_{10} 3 = y \), express the following in terms of \( x \) and \( y \) only:
      i. \( \log_{10} 24 \)
      ii. \( \log_{10} \frac{9}{8} \)
      iii. \( \log_{10} 720 \)

4. (a) The streptococci bacteria population \( N \) at time \( t \) (in months) is given by \( N = N_0 e^{2t} \) where \( N_0 \) is the initial population. If the initial population was 100, how long does it take for the population to reach one million?
   (b) The formula for the amount of energy \( E \) (in joules) released by an earthquake is
      \[ E = 1.74 \times 10^{19} \times 10^{1.44M} \]
      where \( M \) is the magnitude of the earthquake on the Richter scale.
      i. The Newcastle earthquake in 1989 had a magnitude of 5 on the Richter scale. How many joules were released?
      ii. In an earthquake in San Francisco in the 1900’s the amount of energy released was double that of the Newcastle earthquake. What was its Richter magnitude?