3.1 Transformations of Quadratic Functions (pp. 99–106)

Let the graph of $g$ be a translation 1 unit left and 2 units up of the function $f(x) = x^2 + 1$.

Write a rule for $g$:

\[ g(x) = f(x + 1) + 2 \]
\[ = (x + 1)^2 + 2 + 2 \]
\[ = x^2 + 2x + 4 \]

Subtract 1 from the input. Add 2 to the output.

Replace $x$ with $x + 1$ in $g(x)$.

Simplify.

The transformed function is $g(x) = x^2 + 2x + 4$.

Describe the transformation of $f(x) = x^2$ represented by $g$. Then graph each function.

1. $g(x) = (x + 4)^2$
2. $g(x) = (x - 7)^2 + 2$
3. $g(x) = -(x + 2)^2 - 1$

Write a rule for $g$.

4. Let $g$ be a horizontal shrink by a factor of $\frac{2}{3}$, followed by a translation 5 units left and 2 units down of the graph of $f(x) = x^2$.

5. Let $g$ be a translation 2 units left and 3 units up, followed by a reflection in the $y$-axis of the graph of $f(x) = x^2 - 2x$.

3.2 Characteristics of Quadratic Functions (pp. 107–116)

Graph $f(x) = 2x^2 - 8x + 1$. Label the vertex and axis of symmetry.

Step 1 Identify the coefficients: $a = 2$, $b = -8$, $c = 1$. Because $a > 0$, the parabola opens up.

Step 2 Find the vertex. First calculate the $x$-coordinate.

\[ x = \frac{-b}{2a} = -\frac{-8}{2(2)} = 2 \]

Then find the $y$-coordinate of the vertex.

\[ f(2) = 2(2)^2 - 8(2) + 1 = -7 \]

So, the vertex is $(2, -7)$. Plot this point.

Step 3 Draw the axis of symmetry $x = 2$.

Step 4 Identify the $y$-intercept $c$, which is 1. Plot the point $(0, 1)$ and its reflection in the axis of symmetry, $(4, 1)$.

Evaluate the function for another value of $x$, such as $x = 1$.

\[ f(1) = 2(1)^2 - 8(1) + 1 = -5 \]

Plot the point $(1, -5)$ and its reflection in the axis of symmetry, $(3, -5)$.

Step 5 Draw a parabola through the plotted points.

Graph the function. Label the vertex and axis of symmetry. Find the minimum or maximum value of $f$. Describe where the function is increasing and decreasing.

6. $f(x) = 3(x - 1)^2 - 4$
7. $g(x) = -2x^2 + 16x + 3$
8. $h(x) = (x - 3)(x + 7)$
3.3 Focus of a Parabola (pp. 119–126)

a. Identify the focus, directrix, and axis of symmetry of \(8x = y^2\). Graph the equation.

Step 1 Rewrite the equation in standard form.

\[
8x = y^2 \\
\frac{1}{8}x^2
\]

Write the original equation. Divide each side by 8.

Step 2 Identify the focus, directrix, and axis of symmetry. The equation has the form \(x = \frac{1}{4p}y^2\), where \(p = 2\). The focus is \((p, 0)\), or \((2, 0)\). The directrix is \(x = -p\), or \(x = -2\). Because \(y\) is squared, the axis of symmetry is the \(x\)-axis.

Step 3 Use a table of values to graph the equation. Notice that it is easier to substitute \(y\)-values and solve for \(x\).

<table>
<thead>
<tr>
<th>(y)</th>
<th>0</th>
<th>±2</th>
<th>±4</th>
<th>±6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(x)</td>
<td>0</td>
<td>0.5</td>
<td>2</td>
<td>4.5</td>
</tr>
</tbody>
</table>

b. Write an equation of the parabola that opens up, whose vertex \((2, 3)\) is 1 unit from the focus.

Sketch the parabola. The vertex \((h, k)\) is \((2, 3)\), and \(p = 1\). Because the vertex is not at the origin and the axis of symmetry is vertical, the equation has the form

\[y = \frac{1}{4p}(x - h)^2 + k\]

Substitute for \(h, k,\) and \(p\) to write an equation of the parabola.

\[y = \frac{1}{4(1)}(x - 2)^2 + 3 = \frac{1}{4}(x - 2)^2 + 3\]

An equation of the parabola is \(y = \frac{1}{4}(x - 2)^2 + 3\).

9. You can make a solar hot-dog cooker by shaping foil-lined cardboard into a parabolic trough and passing a wire through the focus of each end piece. For the trough shown, how far from the bottom should the wire be placed?

10. Graph the equation \(36y = x^2\). Identify the focus, directrix, and axis of symmetry.

Write an equation of the parabola with the given characteristics.

11. opens to the left; vertex \((0, 0)\) is 2 units from the directrix

12. focus: \((2, 2)\) vertex: \((2, 6)\)
4.3 Completing the Square (pp. 163–170)

Solve \( x^2 + 12x + 8 = 0 \) by completing the square.

\[
x^2 + 12x + 8 = 0
\]
\[
x^2 + 12x = -8
\]
\[
x^2 + 12x + 36 = -8 + 36
\]
\[
(x + 6)^2 = 28
\]
\[
x + 6 = \pm \sqrt{28}
\]
\[
x = -6 \pm \sqrt{28}
\]
\[
x = -6 \pm 2\sqrt{7}
\]

The solutions are \( x = -6 + 2\sqrt{7} \) and \( x = -6 - 2\sqrt{7} \).

12. An employee at a local stadium is launching T-shirts from a T-shirt cannon into the crowd during an intermission of a football game. The height \( h \) (in feet) of the T-shirt after \( t \) seconds can be modeled by \( h = -16t^2 + 96t + 4 \). Find the maximum height of the T-shirt.

Solve the equation by completing the square.

13. \( x^2 + 16x + 17 = 0 \)

14. \( 4x^2 + 16x + 25 = 0 \)

15. \( 9x(x - 6) = 81 \)

16. Write \( y = x^2 - 2x + 20 \) in vertex form. Then identify the vertex.

4.4 Using the Quadratic Formula (pp. 173–182)

Solve \( -x^2 + 4x = 5 \) using the Quadratic Formula.

\[
-x^2 + 4x = 5
\]
\[
-x^2 + 4x - 5 = 0
\]
\[
x = \frac{-4 \pm \sqrt{4^2 - 4(-1)(-5)}}{2(-1)}
\]
\[
x = \frac{-4 \pm \sqrt{-4}}{-2}
\]
\[
x = \frac{-4 \pm 2i}{-2}
\]
\[
x = 2 \pm i
\]

The solutions are \( 2 + i \) and \( 2 - i \).

Solve the equation using the Quadratic Formula.

17. \( -x^2 + 5x = 2 \)

18. \( 2x^2 + 5x = 3 \)

19. \( 3x^2 - 12x + 13 = 0 \)

Find the discriminant of the quadratic equation and describe the number and type of solutions of the equation.

20. \( -x^2 - 6x - 9 = 0 \)

21. \( x^2 - 2x - 9 = 0 \)

22. \( x^2 + 6x + 5 = 0 \)
#6: Decrease: \((-\infty, 1)\)
Increase: \((1, \infty)\)

#7: Increase: \((-\infty, 4)\)
Decrease: \((4, \infty)\)

#8: Decrease: \((-\infty, -2)\)
Increase: \((-2, \infty)\)
Detailed explanation for p.136 #12

p.136 #12 Focus (2,2) Vertex (2,6)

Directrix

Make a sketch:

Compare the locations of the vertex and focus. The focus is below the vertex. Since the parabola cannot intersect the focus or the directrix, the parabola must open downward.

So, the equation to use is

\[ y = \frac{1}{4p} (x-h)^2 + k \]  (Since the parabola opens downward, \( \frac{1}{4p} \) must be negative in the final answer)

The focus is 4 units away from the vertex, so \( p = 4 \). The vertex is (2,6), so \( h = 2 \) and \( k = 6 \). Substitute these values into the equation:

\[ y = \frac{1}{4 \cdot 4} (x-2)^2 + 6 \]

If you have to find the directrix, remember that it is also \( p \) units from the vertex on the other side of the focus, so

the equation of the directrix here is \( y = 10 \). The axis of symmetry here is \( x = 2 \).
1. The graph is a translation 4 units left of the parent quadratic function.

\[ g(x) = (x + 4)^2 \]

2. The graph is a translation 7 units right and 2 units up of the parent quadratic function.

\[ g(x) = (x - 7)^2 + 2 \]

3. The graph is a vertical stretch by a factor of 3 followed by a reflection in the x-axis and a translation 2 units left and 1 unit down.

\[ g(x) = -3(x + 4)^2 - 1 \]

4. \[ g(x) = \frac{9}{4}(x + 5)^2 - 2 \]
3.4 Maintaining Mathematical Proficiency

38. \((x + 3)(x + 1)\)  
40. \(3(x - 4)(x - 1)\)

39. \((x - 2)(x - 1)\)  
41. \(5(x + 3)(x - 2)\)

Chapter 3 Review

5. \(g(x) = (-x + 2)^2 - 2(-x + 2) + 3 = x^2 - 2x + 3\)

6. The minimum value is \(-4\); The function is decreasing to the left of \(x = 1\) and increasing to the right of \(x = 1\).

7. The maximum value is \(35\); The function is increasing to the left of \(x = 4\) and decreasing to the right of \(x = 4\).

8. The minimum value is \(-25\); The function is decreasing to the left of \(x = -2\) and increasing to the right of \(x = -2\).
ANSWERS

9. 2.25 in.

10. The focus is \((0, 9)\), the directrix is \(y = -9\), and the axis of symmetry is \(x = 0\).

11. \(x = -\frac{1}{8}y^2\)

12. \(y = -\frac{1}{16}(x - 2)^2 + 6\)
12. 148 ft

13. $x = -8 \pm \sqrt{47}$

14. $x = \frac{-4 \pm 3i}{2}$

15. $x = 3 \pm 3\sqrt{2}$

16. $y = (x - 1)^2 + 19; (1, 19)$