Review: Graphing & Transforming Cube and Cube Root functions + Circles

Write a rule for $g$ that represents the indicated transformations of the graph of $f$.

1. $f(x) = x^3$; translation 1 unit left, followed by a reflection in the $y$-axis
   
   $g(x) = $ ______________________

2. $f(x) = \sqrt[3]{x}$; vertical stretch by a factor of 2, followed by a translation 3 units left
   
   $g(x) = $ ______________________

3. $f(x) = x^3$; horizontal shrink by a factor of $\frac{1}{3}$ and a translation 4 units up, followed by a reflection in the $x$-axis
   
   $g(x) = $ ______________________

Graph the function. Identify the $x$-intercepts and the points where the local maximums and local minimums occur. Determine the intervals for which the function is increasing or decreasing and the domain and range.

4. $f(x) = x^3 - 2x^2 - x + 1$

   ![Graph of $f(x)$]

   x-intercept(s): ______________________
   Increasing: ___________________ Decreasing: ____________________
   Local Max: ___________________ Local Min: ___________________
   Domain: _____________________ Range: _____________________

Describe the transformation of $f$ represented by $g$. Then graph each function.

5. $f(x) = \sqrt[3]{x}$, $g(x) = \sqrt[3]{x - 3} + 4$

   Transformation: ______________________
6. \( f(x) = \sqrt[3]{x} \), \( g(x) = -\sqrt[3]{x} - 2 \)

Transformation: _______________________________

7. \( f(x) = x^3 \), \( g(x) = \frac{1}{3} (x + 3)^3 \)

Transformation: _______________________________

8. \( f(x) = \sqrt[3]{x} \), \( g(x) = \sqrt[3]{-32x - 4} \)

Transformation: _______________________________

Sketch a graph of the polynomial function \( f \) having the given characteristics. Use the graph to describe the degree and leading coefficient of the function \( f \).

9. • \( f \) is decreasing on the intervals \((-\infty, -1)\) and \((3, \infty)\);
   \( f \) is increasing on the interval \((-1, 3)\).
   • \( f(x) > 0 \) on the intervals \((-\infty, -2)\) and \((0, 5)\);
   \( f(x) < 0 \) on the intervals \((-2, 0)\) and \((5, \infty)\).

Write the equation of the circle in standard form.

10. \( x^2 + y^2 + 8x - 2y - 64 = 0 \)
11. \(x^2 + y^2 + 24x + 6y + 137 = 0\)

Explain how you would determine whether the function is \textit{even}, \textit{odd}, or \textit{neither}.

12. \(g(x) = 3x^6 - 2x^2 - 8\)

13. \(f(x) = -5x^5 - 3x^2\)

14. Find the center and radius of the circle with equation \((x + 9)^2 + (y + 5)^2 = 64\).

15. Write the standard equation of the circle in the graph.