Chapter 3 Section 1: Transformations of Quadratic Functions

- A quadratic function is a function that can be written in the form \( f(x) = \_\_\_\_\_\_\_\_\_\_\_\_\_ \) where \( a \neq 0 \).

**Horizontal Translations:**

\[
f(x) = x^2 \\
f(x-h) = (x-h)^2
\]

h > 0, shift right 

h < 0, shift left 

**Example 1:** Describe the transformation of \( f(x) = x^2 \) represented by \( h(x) = (x-1)^2 + 2 \).

**Vertical Translations:**

\[
f(x) = x^2 \\
f(x) + k = x^2 + k
\]

k > 0, shift up 

k < 0, shift down 

**Reflection in the x-axis**

\[
f(x) = x^2 \\
-f(x) = -(x^2) = -x^2
\]

flips over the x-axis

**Reflection in the y-axis**

\[
f(x) = x^2 \\
f(-x) = (-x)^2 = x^2
\]

\( y - x^2 \) is its own reflection in the y-axis.

**Horizontal stretch and shrink**

\[
f(x) = x^2 \\
f(ax) = (ax)^2
\]

- horizontal stretch (away from y-axis) when \( 0 < a < 1 \)
- horizontal shrink (toward y-axis) when \( a > 1 \)

**Vertical stretch and shrink**

\[
f(x) = x^2 \\
a \cdot f(x) = ax^2
\]

- vertical stretch (away from x-axis) when \( 0 < a < 1 \)
- vertical shrink (toward x-axis) when \( a > 1 \)
Example 2: Describe the transformation of \( f(x) = x^2 \) represented by \( h \).

a) \( h(x) = -3x^2 \)

b) \( h(x) = \left(\frac{1}{4}x\right)^2 - 2 \)

Writing Transformations of Quadratic Functions

- The vertex form of a quadratic function is \( f(x) = \text{_______________________________} \), where \( a \neq 0 \) and the vertex is \((h,k)\).

Example 3: Let the graph of \( g \) be a horizontal shrink by a factor of \( 1/3 \) and a reflection in the \( y \)-axis, followed by a translation 2 units up of the graph of \( f(x) = x^2 \). Write a rule for \( g \) and identify the vertex.

Example 4: Let the graph of \( g \) be a translation 4 units left and 1 unit down, followed by a reflection in the \( y \)-axis of the graph \( f(x) = x^2 \). Write a rule for \( g \) and identify the vertex.