7-7 Word Problem Practice

Base e and Natural Logarithms

1. INTEREST Horatio opens a bank account that pays 2.3% annual interest compounded continuously. He makes an initial deposit of 10,000. What will be the balance of the account in 10 years? Assume that he makes no additional deposits and no withdrawals.

2. INTEREST Janie’s bank pays 2.8% annual interest compounded continuously on savings accounts. She placed $2000 in the account. How long will it take for her initial deposit to double in value? Assume that she makes no additional deposits and no withdrawals. Round your answer to the nearest quarter year.

3. POPULATION The equation $A = A_0e^{rt}$ describes the growth of the world’s population where $A$ is the population at time $t$, $A_0$ is the population at $t = 0$, and $r$ is the annual growth rate. The world’s population at the start of 2008 was estimated at 6,641,000,000. If the annual growth rate is 1.2%, when will the world population reach 9 billion?

4. BACTERIA A bacterial population grows exponentially, doubling every 72 hours.

<table>
<thead>
<tr>
<th>bacteria</th>
<th>$x$</th>
<th>$2x$</th>
<th>$4x$</th>
<th>$8x$</th>
</tr>
</thead>
<tbody>
<tr>
<td>time</td>
<td>0</td>
<td>72</td>
<td>144</td>
<td>216</td>
</tr>
</tbody>
</table>

Let $P$ be the initial population size and let $t$ be time in hours. Use the equation from Exercise 3 to write a formula using the natural base exponential function that gives the size of the population $y$ as a function of $P$ and $t$.

5. MONEY MANAGEMENT Linda wants to invest $20,000. She is looking at two possible accounts. Account A is a standard savings account that pays 3.4% annual interest compounded continuously. Account B would pay her a fixed amount of $200 every quarter.

   a. If Linda can invest the money for 5 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?

   b. If Linda can invest the money for 10 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?

   c. If Linda can invest the money for 20 years only, which account would give her the higher return on her investment? How much more money would she make by choosing the higher paying account?
Approximations for $\pi$ and $e$

The following expression can be used to approximate $e$. If greater and greater values of $n$ are used, the value of the expression approximates $e$ more and more closely.

$$\left(1 + \frac{1}{n}\right)^n$$

Another way to approximate $e$ is to use this infinite sum. The greater the value of $n$, the closer the approximation.

$$e = 1 + \frac{1}{1} + \frac{1}{2 \cdot 3} + \frac{1}{2 \cdot 3 \cdot 4} + \ldots + \frac{1}{2 \cdot 3 \cdot 4 \ldots n} + \ldots$$

In a similar manner, $\pi$ can be approximated using an infinite product discovered by the English mathematician John Wallis (1616–1703).

$$\frac{\pi}{2} = \frac{2}{1} \cdot \frac{2}{3} \cdot \frac{4}{3} \cdot \frac{4}{5} \cdot \frac{6}{5} \cdot \frac{6}{7} \cdot \ldots \cdot \frac{2n}{2n - 1} \cdot \frac{2n}{2n + 1} \ldots$$

Solve each problem.

1. Use a calculator with an $e^x$ key to find $e$ to 7 decimal places.

2. Use the expression $\left(1 + \frac{1}{n}\right)^n$ to approximate $e$ to 3 decimal places. Use 5, 100, 500, and 7000 as values of $n$.

3. Use the infinite sum to approximate $e$ to 3 decimal places. Use the whole numbers from 3 through 6 as values of $n$.

4. Which approximation method approaches the value of $e$ more quickly?

5. Use a calculator with a $\pi$ key to find $\pi$ to 7 decimal places.

6. Use the infinite product to approximate $\pi$ to 3 decimal places. Use the whole numbers from 3 through 6 as values of $n$.

7. Does the infinite product give good approximations for $\pi$ quickly?

8. Show that $\pi^4 + \pi^6$ is equal to $e^6$ to 4 decimal places.

9. Which is greater, $e^\pi$ or $\pi^e$?

10. The expression $x^{\frac{1}{x}}$ reaches a maximum value at $x = e$. Use this fact to prove the inequality you found in Exercise 9.