8.1 Practice

In Exercises 1–6, tell whether x and y show direct variation, inverse variation, or neither.

1. \( y = \frac{12}{x} \)  
2. \( xy = 15 \)  
3. \( 9x = y \)

4. \( y = x - 3 \)  
5. \( \frac{y}{x} = 9 \)  
6. \( xy = \frac{1}{3} \)

In Exercises 7–10, tell whether x and y show direct variation, inverse variation, or neither.

7. \[
\begin{array}{cccc}
 x & 2.5 & 4 & 7.5 \\
 y & 30 & 48 & 90 \\
\end{array}
\]

8. \[
\begin{array}{cccc}
 x & 12 & 5 & 2.5 & 1.5 \\
 y & 35 & 84 & 168 & 280 \\
\end{array}
\]

9. \[
\begin{array}{cccc}
 x & 2.5 & 3 & 6 & 10 \\
 y & 8 & 9.6 & 1.6 & 6 \\
\end{array}
\]

10. \[
\begin{array}{cccc}
 x & 2.5 & 10 & 16 & 21 \\
 y & 672 & 168 & 105 & 80 \\
\end{array}
\]

In Exercises 11–13, the variables x and y vary inversely. Use the given values to write an equation relating x and y. Then find y when \( x = 3 \).

11. \( x = 4, y = -3 \)  
12. \( x = \frac{2}{3}, y = -5 \)  
13. \( x = -10, y = \frac{-1}{5} \)

14. The variables x and y vary inversely. Describe and correct the error in writing an equation relating x and y.

\[
\begin{array}{cccc}
 x & \frac{1}{3}, y = 2 \\
 xy & = a \\
 \frac{1}{3} \times 2 & = a \\
 a & = \frac{2}{3} \\
 So, y & = \frac{3x}{2}.
\end{array}
\]

\[
\begin{array}{cccc}
 x & 6, y = 5 \\
 \frac{y}{x} & = a \leftarrow \text{Wrong equation} \\
 \frac{5}{6} & = a \\
 \text{So, } y & = \frac{5}{6x} \\

\text{Should be:} & \frac{y}{x} = \frac{5}{6} \times 2 = xy.
\end{array}
\]

\[
\begin{array}{cccc}
 x & 6, y = 5 \\
 \frac{y}{x} & = a \leftarrow \text{Wrong equation} \\
 \frac{5}{6} & = a \\
 \text{So, } y & = \frac{5}{6x} \\

\text{Should be:} & \frac{y}{x} = \frac{5}{6} \times 2 = xy.
\end{array}
\]

\[
\begin{array}{cccc}
 x & 6, y = 5 \\
 \frac{y}{x} & = a \leftarrow \text{Wrong equation} \\
 \frac{5}{6} & = a \\
 \text{So, } y & = \frac{5}{6x} \\

\text{Should be:} & \frac{y}{x} = \frac{5}{6} \times 2 = xy.
\end{array}
\]
15. The current $y$ in a certain circuit varies inversely with the resistance $x$ in the circuit. If the current is 8 amperes when the resistance is 20 ohms, what will the current be when the resistance increases to 25 ohms?

$$6.4 \text{ Amp}
$\text{ers}$$

**Example:** Suppose $y$ varies directly with $x$ and $w$ but varies inversely as the square of $z$. Find the equation of variation if $y = 100$ when $x = 2$, $w = 4$, and $z = 20$.

**Solution:**

$$y = \frac{axw}{z^2}$$

Given the direct variation with $y$, constant $a$, and $x$ and $w$ are in the numerators. Given the inverse variation with $y$, $z^2$ is in the denominator.

$$100 = \frac{a(2)(4)}{(20)^2}$$

Substitute.

$$a = 5000$$

Solve for $a$.

$$y = \frac{5000xw}{z^2}$$

Now you can solve for one of the variables if you were given the values of the other three variables.

**In Exercises 16–19, solve the combined variation problems.**

16. Suppose $x$ varies directly with $y$ and the square root of $z$. When $x = -18$ and $y = 2$, $z = 9$. Find $y$ when $x = 10$ and $z = 4$.

$$y = \frac{5}{3}$$

17. Suppose $w$ varies inversely with $z$ and the cube root of $v$, but varies directly with $y$. When $w = 4$, $v = 27$, and $y = 2$, $z = 5$. Find $w$ when $y = 3$, $v = 64$, and $z = 6$.

$$w = \frac{45}{4} \text{ or } 3.75$$

18. The volume $V$ of wood in a tree varies directly with the height $h$ and inversely with the square of the girth $g$. The volume of a tree is 144 cubic meters when the height is 20 meters and the girth is 1.5 meters. What is the height of a tree with a volume of 100 cubic meters and girth of 2 meters?

$$h = 24.69$$

19. The pressure $P$ of a gas varies directly with the number of moles $n$ and temperature $T$ of the gas and inversely with volume $V$. Given the equation of the situation described above, as the pressure of a gas increases, what happens to the volume of the gas? Now write the number of moles as a function of pressure, volume, and temperature.

**Volume goes down as pressure goes up.**

**Isolate $n$:**

$$n = \frac{P V}{k T}$$
Work for #15:

1. "Varies inversely" means \( y = \frac{k}{x} \) in the general equation.

Since "y" comes first in the sentence, we use "y" first in the equation: \( y = \frac{k}{x} \)

2. Find "k" based on the given information:

\[
y = \frac{k}{x} \quad \Rightarrow \quad 8 = \frac{k}{20} \quad \Rightarrow \quad k = 160
\]

3. Write the equation with value for "k"

\[
y = \frac{160}{x}
\]

4. Substitute new values of variables and simplify.

(Remember: y represents the current and \( x \) represents the resistance)

\[
y = \frac{160}{25} = \frac{32}{5} \quad \text{or} \quad 6.4 \text{ Amperes}
\]

Work for #12:

1. "Varies inversely" means \( y = \frac{k}{x} \)

2. Use given \( x \) and \( y \) to find \( k \): \(-5 = \frac{k}{\frac{2}{3}} \rightarrow k = -10 \times \frac{3}{2} \)

3. Write \( y = \frac{k}{x} \) using the value of \( k \):

\[
y = \frac{-10 \times \frac{3}{2}}{x} \rightarrow y = \frac{-10}{3x}
\]

4. "Find \( y \) when \( x = 3 \)." Use this equation containing "k".

\[
y = \frac{-10}{3 \times 3} \Rightarrow y = \frac{-10}{9} \Rightarrow y = \frac{-10}{9}
\]
17. Suppose \( w \) varies inversely with \( z \) and the cube root of \( v \), but varies directly with \( y \).
When \( w = 4, \ v = 27, \) and \( y = 2, \ z = 5 \). Find \( w \) when \( y = 3, \ v = 64, \) and \( z = 6 \).

Work for #17:

1. Set up equation: \( w = k \cdot \frac{y}{\sqrt[3]{v} \cdot \sqrt{z}} \) \( \text{"Varies directly" means multiply by the } \ "k" \text{ constant} \)
   \( \text{"Varies inversely" means divide by the } \ "k" \text{ constant} \)

2. Find \( k \) based on given information:

\[
\begin{align*}
  w &= \frac{ky}{z \cdot \sqrt[3]{v}} \\
  4 &= \frac{k \cdot 2}{5 \cdot \sqrt[3]{27}} \\
  4 &= \frac{k \cdot 2}{5 \cdot 3} \\
  4 &= \frac{2k}{15} \\
  15 \cdot 4 &= 2k \\
  30 &= k
\end{align*}
\]

3. Write equation with new value for \( k \):

\[
W = \frac{k \cdot y}{z \cdot \sqrt[3]{v}} \\
W = \frac{30y}{z \cdot \sqrt[3]{v}}
\]

4. Substitute new values of variables and simplify:

\[
W = \frac{30 \cdot 3}{6 \cdot \sqrt[3]{64}} \\
W = \frac{30 \cdot 3}{6 \cdot 4} \\
W = \frac{15}{4} \text{ or } 3.75
\]