7.6 Enrichment and Extension

Solving Exponential and Logarithmic Equations

The natural base $e$ is used to solve the equation that models Newton’s Law of Cooling. This law was explored for Example 2 of the lesson.

$$T = (T_0 - T_R)e^{-rt} + T_R$$

where $T$ is the temperature after $t$ minutes, $T_0$ is the initial temperature, $T_R$ is the surrounding temperature, $t$ represents time (in minutes), and $r$ represents the cooling rate.

**Example:** When the constant surrounding temperature is $65^\circ F$, an object will cool from $160^\circ F$ to $120^\circ F$ in 30 minutes. How long will it take to cool the object to a temperature of $70^\circ F$?

$$120 = (160 - 65)e^{-r \cdot 30} + 65$$

Substitute the given values into the function.

$$\frac{55}{95} = e^{-r \cdot 30}$$

Simplify to solve for $r$.

$$\ln\left(\frac{95}{55}\right) = \ln(e^{-r \cdot 30}) \rightarrow 0.5465 = 30r \rightarrow r = 0.0182$$

Use $r$ to determine when the temperature is $70^\circ F$.

$$70 = 65 + (160 - 65)e^{-0.0182t}$$

In Exercises 1 and 2, use $T = (T_0 - T_R)e^{-rt} + T_R$.

1. At a local restaurant, the cook prepares enough soup at night so that there is plenty of soup for customers the next day. Refrigeration is necessary, but the soup is too hot at $220^\circ F$ to put directly into the fridge. It needs to be no more than $70^\circ F$. The cook cooled the soup by placing it in a sink of water constantly running at $40^\circ F$. After 10 minutes, the soup had cooled down to $140^\circ F$. How long will it take to be able to place the soup in the fridge?

2. Milk is taken out of the fridge and left on a table. Its temperature is $35^\circ F$ and the house is a constant $70^\circ F$. The temperature of the milk rose $5^\circ F$ after 1 hour. Milk is unsafe to consume at temperatures over $45^\circ F$. How long can the milk be left out and still be safe to consume?